

On Soft Gluon Effects in Deep-Inelastic Structure Functions

A. Vogt

*Instituut-Lorentz, University of Leiden
P.O. Box 9506, 2300 RA Leiden, The Netherlands*

Abstract

The behaviour of the quark coefficient functions in deep-inelastic scattering is investigated for large values of the Bjorken variable x . By combining results of soft-gluon resummation and fixed-order calculations, we determine the coefficients of the four leading large- x logarithms, $\alpha_s^k [\{\ln(1-x)\}^{2k-l}/(1-x)]_+$, $l = 1, \dots, 4$, to all orders in the strong coupling constant α_s . This result includes two more terms for the three-loop coefficient functions than previously specified in the literature. The effect of the fifth logarithmic contribution is approximately evaluated. The terms derived here are required, but also seem to be sufficient, for a reliable representation of the coefficient functions at large x .

Knowledge of the proton's quark distributions at large momentum fractions x is relevant to the search for new phenomena at high-energy colliders like HERA, TEVATRON and the future LHC. These distributions are usually inferred from data on structure functions in deep-inelastic scattering (DIS). For instance, data even up to $x = 0.98$ have recently been employed [1] to scrutinize a possible non-standard behaviour of the large- x quark distributions [2] suggested in connection with the initial HERA high- Q^2 anomaly [3]. In perturbative QCD the link between the quark distributions and the structure functions $F_i(x, Q^2)$, $i = 1, 2, 3$, is given by the coefficient functions¹

$$C_{i,q}(x, Q^2) = \delta(1-x) + \sum_{k=1} a_s^k c_{i,q}^{(k)}(x) \quad (1)$$

with $a_s = \alpha_s(Q^2)/(4\pi)$. The expansion coefficients $c_{i,q}^{(k)}(x)$ and their Mellin- N space counterparts $c_{i,q}^{(k)N}$ contain universal (i.e., i -independent) soft-gluon contributions of the form

$$\left[\frac{\ln^{l-1}(1-x)}{1-x} \right]_+ \longleftrightarrow \frac{(-1)^l}{l} (\ln^l N + \text{subleading terms}) , \quad (2)$$

$l = 1, \dots, 2k$. At sufficiently large x / large N , these terms spoil the convergence of finite-order approximations to $C_{i,q}$ and need to be resummed to all orders. It is thus important to derive a reliable estimate of their impact at the x -values included in data analyses.

The soft-gluon resummation has been carried out for the leading and next-to-leading logarithms (in the sense of eq. (8) below) some time ago [4, 5]. In this letter we combine these results with terms from the one- and two-loop coefficient functions $c_{i,q}^{(1,2)}(x)$ calculated in refs. [6, 7]. This information fixes the first four towers, $j = 0, 1, 2$ and 3, of large- x logarithms, i.e., the coefficients of $\ln^{2k-j} N$ in $c_{i,q}^{(k)N}$ at each order k . It also facilitates an estimate of the effect of the fifth tower, $j = 4$. As shown below, these four to five towers are required, but also seem to be sufficient, for a realistic estimate of the higher-order effects at large x .

Up to terms which vanish for $N \rightarrow \infty$, the N -space coefficient functions take the form

$$C_{i,q}^N(Q^2) = (1 + a_s g_{01} + a_s^2 g_{02} + \dots) \cdot \exp [G^N(Q^2)] . \quad (3)$$

The first factor collects contributions which are constant for $N \rightarrow \infty$. Adopting the $\overline{\text{MS}}$ scheme, the first-order term reads [6]

$$g_{01} = (-9 - 2\zeta_2 + 2\gamma_e^2 + 3\gamma_e) C_F . \quad (4)$$

Here ζ_l stands for Riemann's ζ -function, γ_e is the Euler-Mascheroni constant, and $C_F = 4/3$ in QCD. The corresponding second-order term can be readily inferred from ref. [7].

¹The renormalization and factorization scales are chosen as $\mu_r^2 = \mu_f^2 = Q^2$ throughout this paper. The additional terms arising for other choices can be deduced from renormalization-group constraints.

The function $G^N(Q^2)$ is the object of the soft-gluon resummation. It is given by [4, 5]

$$G^N(Q^2) = \int_0^1 dz \frac{z^{N-1} - 1}{1 - z} \left[\int_{Q^2}^{(1-z)Q^2} \frac{dq^2}{q^2} A(a_s(q^2)) + \frac{1}{2} B(a_s([1-z]Q^2)) \right] + \mathcal{O}(a_s(a_s \ln N)^k) \quad (5)$$

with $A(a_s) = 4 C_F a_s + 8 C_F K a_s^2 + \dots$, $B = -6 C_F a_s + \dots$ and

$$K = \left(\frac{67}{18} - \zeta_2 \right) C_A - \frac{5}{9} N_f . \quad (6)$$

$C_A = 3$ in QCD, and N_f denotes the number of effectively massless quark flavours. These terms of $A(a_s)$ and $B(a_s)$, together with the lowest two coefficients of the β -function,

$$\beta_0 = \frac{11}{3} C_A - \frac{2}{3} N_f , \quad \beta_1 = \frac{34}{3} C_A^2 - \frac{10}{3} C_A N_f - 2 C_F N_f \quad (7)$$

are sufficient for deriving the next-to-leading logarithms.

The integrals occurring in eq. (5) can be found in ref. [8]. Using the abbreviations $L \equiv \ln N$ and $\lambda \equiv \beta_0 a_s L$, the result can be written as

$$G^N(Q^2) = L g_1(a_s L) + g_2(a_s L) + a_s g_3(a_s L) + \dots \quad (8)$$

with

$$\begin{aligned} g_1(a_s L) &= \frac{4 C_F}{\beta_0 \lambda} \left[\lambda + (1 - \lambda) \ln(1 - \lambda) \right] \equiv \sum_{k=1} g_{1k}(a_s L)^k \\ &= \sum_{k=1}^{\infty} \frac{4 C_F \beta_0^{k-1}}{k(k+1)} (a_s L)^k , \end{aligned} \quad (9)$$

$$\begin{aligned} g_2(a_s L) &= -\frac{C_F(3 + 4\gamma_e)}{\beta_0} \ln(1 - \lambda) - \frac{8 C_F K}{\beta_0^2} \left[\lambda + \ln(1 - \lambda) \right] \\ &\quad + \frac{4 C_F \beta_1}{\beta_0^3} \left[\lambda + \ln(1 - \lambda) + \frac{1}{2} \ln^2(1 - \lambda) \right] \equiv \sum_{k=1} g_{2k}(a_s L)^k \\ &= \sum_{k=1}^{\infty} \left\{ \frac{3 + 4\gamma_e}{\beta_0} + \theta_{k2} \left(\frac{8K}{\beta_0^2} + \frac{4\beta_1}{\beta_0^3} [S_1(k-1) - 1] \right) \right\} \frac{C_F \beta_0^k}{k} (a_s L)^k . \end{aligned} \quad (10)$$

Here $\theta_{kj} = 1$ for $k \geq j$ and $\theta_{kj} = 0$ else, and $S_1(k) = \sum_{j=1}^k 1/j$. The function g_3 in eq. (8),

$$g_3(a_s L) \equiv \sum_{k=2} g_{3k}(a_s L)^{k-1} , \quad (11)$$

is presently unknown except for its leading term which can be determined from ref. [7].

Its coefficient g_{32} reads

$$\begin{aligned} g_{32} &= \left(\frac{3155}{54} - 40 \zeta_3 - \frac{22}{3} \zeta_2 - 8 \zeta_2 \gamma_e + \frac{22}{3} \gamma_e^2 + \frac{367}{9} \gamma_e \right) C_F C_A \\ &\quad \left(\frac{3}{2} + 24 \zeta_3 - 12 \zeta_2 \right) C_F^2 + \left(-\frac{247}{27} + \frac{4}{3} \zeta_2 - \frac{4}{3} \gamma_e^2 - \frac{58}{9} \gamma_e \right) C_F N_f . \end{aligned} \quad (12)$$

We are now ready to evaluate the resummed large- N coefficient function (3). Expansion of the exponential yields

$$C_{i,q}^N(Q^2) = 1 + \sum_{k=1}^{\infty} a_s^k \left(c_{k1} L^{2k} + c_{k2} L^{2k-1} + c_{k3} L^{2k-2} + c_{k4} L^{2k-3} + \mathcal{O}(L^{2k-4}) \right) \quad (13)$$

with

$$\begin{aligned} c_{k1} &= \frac{g_{11}^k}{k!} \\ c_{k2} &= \frac{g_{11}^{k-1}}{(k-1)!} g_{21} + \frac{\theta_{k2} g_{11}^{k-2}}{(k-2)!} g_{12} \\ c_{k3} &= \frac{g_{11}^{k-1}}{(k-1)!} g_{01} + \frac{\theta_{k2} g_{11}^{k-2}}{(k-2)!} \left(g_{22} + \frac{1}{2} g_{21}^2 \right) + \frac{\theta_{k3} g_{11}^{k-3}}{(k-3)!} (g_{13} + g_{12} g_{21}) + \frac{\theta_{k4} g_{11}^{k-4}}{2(k-4)!} g_{12}^2 \\ c_{k4} &= \frac{\theta_{k2} g_{11}^{k-2}}{(k-2)!} (g_{21} g_{01} + g_{32}) + \frac{\theta_{k3} g_{11}^{k-3}}{(k-3)!} (g_{12} g_{01} + g_{23} + g_{22} g_{21} + \frac{1}{6} g_{21}^3) \\ &\quad + \frac{\theta_{k4} g_{11}^{k-4}}{(k-4)!} (g_{14} + g_{13} g_{21} + g_{12} g_{22} + \frac{1}{2} g_{12} g_{21}^2) + \frac{\theta_{k5} g_{11}^{k-5}}{(k-5)!} (g_{13} g_{12} + \frac{1}{2} g_{12}^2 g_{21}) \\ &\quad + \frac{\theta_{k6} g_{11}^{k-6}}{6(k-6)!} g_{12}^3 . \end{aligned} \quad (14)$$

Inspection of eq. (14) reveals that the coefficients up to c_{k4} are indeed fixed by eqs. (4), (9), (10) and (12).² Especially the four leading large- x terms of the three-loop ($k=3$) coefficient functions $c_{i,q}^{(3)}$ are thus determined. Transformation to x -space [9] yields

$$\begin{aligned} c_{i,q}^{(3)}(x) &= 8 C_F^3 \left[\frac{\ln^5(1-x)}{1-x} \right]_+ - \left(30 C_F^3 + \frac{220}{9} C_F^2 C_A - \frac{40}{9} C_F^2 N_f \right) \left[\frac{\ln^4(1-x)}{1-x} \right]_+ \\ &\quad + \left\{ \left(-36 - 96 \zeta_2 \right) C_F^3 + \left(\frac{1732}{9} - 32 \zeta_2 \right) C_F^2 C_A + \frac{484}{27} C_F C_A^2 \right. \\ &\quad \left. + \frac{16}{27} C_F N_f^2 - \frac{176}{27} C_F C_A N_f - \frac{280}{9} C_F^2 N_f \right\} \left[\frac{\ln^3(1-x)}{1-x} \right]_+ \\ &\quad + \left\{ \left(\frac{279}{2} + 16 \zeta_3 + 288 \zeta_2 \right) C_F^3 + \left(-\frac{8425}{18} + 240 \zeta_3 + \frac{724}{3} \zeta_2 \right) C_F^2 C_A \right. \\ &\quad \left. + \left(-\frac{4649}{27} + \frac{88}{3} \zeta_2 \right) C_F C_A^2 + \left(\frac{683}{9} - \frac{112}{3} \zeta_2 \right) C_F^2 N_f \right. \\ &\quad \left. + \left(\frac{1552}{27} - \frac{16}{3} \zeta_2 \right) C_F C_A N_f - \frac{116}{27} C_F N_f^2 \right\} \left[\frac{\ln^2(1-x)}{1-x} \right]_+ + \dots \quad (15) \end{aligned}$$

The first two terms of eq. (15) have already been given in ref. [11]. Besides as a useful cross check for a future exact calculation of $c_{i,q}^{(3)}(x)$, the above result can also be used for improved approximate reconstructions [10] of this function along the lines of refs. [11, 12].

²The agreement of the predictions (14) for c_{21} , c_{22} and c_{23} with the results of the full $\mathcal{O}(a_s^2)$ calculation [7] provides a non-trivial check of the resummation (8)–(10) of the next-to-leading logarithms.

The corresponding higher-order results are straightforward if cumbersome. The numerical values of the coefficients c_{kl} in eq. (14) are presented in table 1 for $N_f = 4$ and $k \leq 10$. In contrast to g_2 in eq. (10) which exhibits a pole at $N = \exp[1/(\beta_0 a_s)]$, the sum (13) converges for all N as long as the number of towers included is finite.

k	c_{k1}	c_{k2}	c_{k3}	c_{k4}	c_{k5}
1	2.66667	7.0785	—	—	—
2	3.55556	26.2834	40.760	-67.13	—
3	3.16049	44.9210	238.885	470.82	$g_{33} - 1235.23$
4	2.10700	48.7090	477.854	2429.46	$c_{11} g_{33} + 3600.12$
5	1.12373	38.3254	581.518	5015.18	$c_{21} g_{33} + 22963.9$
6	0.49944	23.5617	505.972	6432.95	$c_{31} g_{33} + 50185.0$
7	0.19026	11.8592	340.954	5933.61	$c_{41} g_{33} + 67307.5$
8	0.06342	5.0464	186.822	4244.45	$c_{51} g_{33} + 64858.9$
9	0.01879	1.8583	86.041	2467.72	$c_{61} g_{33} + 48498.6$
10	0.00501	0.6028	34.118	1204.34	$c_{71} g_{33} + 29487.7$

Table 1: Numerical values of the coefficients c_{kl} in eq. (14) for $N_f = 4$. Also shown are the corresponding (incomplete) results c_{k5} for the fifth tower of logarithms in eq. (13).

Before turning to the effect of these higher-order contributions, it is instructive to compare the α_s^2 and α_s^3 parts of eq. (13) with the respective exact and approximate results for $c_{2,q}^{(2)}$ [7] and $c_{2,q}^{(3)}$ [11, 13].³ For this purpose the coefficient function is convoluted (for $\alpha_s = 0.2$, a value typical for scales probed in fixed-target DIS) with a simple, but characteristic model $f(x)$ of the large- x quark distributions. The second-order comparison is shown in fig. 1, its third-order counterpart in fig. 2. Two and three terms in the large- N expansion (13) (i.e., $\ln^4 N$ and $\ln^3 N$ at two-loop and $\ln^6 N$, $\ln^5 N$, and $\ln^4 N$ at three-loop) turn out to be sufficient for good approximations to $c_{2,q}^{(3)} \otimes f$ and $c_{3,q}^{(3)} \otimes f$, respectively, at large x . It should be noted that it is essential for this fast convergence to use the expansion in N -space. For instance, $c_{2,q}^{(2)} \otimes f$ is severely overestimated if only the two leading x -space terms, $[\ln^3(1-x)/(1-x)]_+$ and $[\ln^2(1-x)/(1-x)]_+$, are kept [11]. In the all-order case, the problem of the x -space expansion has been elucidated in ref. [14].

The higher-order soft-gluon corrections, $k \geq 4$ in eq. (13), are illustrated in the same manner in fig. 3. Under these conditions the terms up to $k = 8$ are sufficient for an accurate representation up to $z = 0.99$. As above the Mellin inversions are performed using the

³These approximations are based on the first five even-integer moments of $c_{2,q}^{(3)}(x)$ [11], supplemented by the first two coefficients in eq. (15). $c_{2,q}^{(3)}(x)$ is rather tightly constrained at $x \gtrsim 0.5$ by this information.

standard contour [15] (see also the discussion of the infinite-order limit in ref. [14])

$$\begin{aligned}
A(x) &= \frac{1}{2\pi i} \left\{ \int_{c-(i+\delta)\infty}^c + \int_c^{c+(i-\delta)\infty} \right\} dN x^{-N} A_N \\
&= \frac{1}{\pi} \int_0^\infty dz \operatorname{Im} \left[\exp(i\phi) x^{-c-z \exp(i\phi)} A_{N=c+z \exp(i\phi)} \right]
\end{aligned} \tag{16}$$

using $c = 2$ and $\phi = 3/4\pi$ corresponding to $\delta = 1$. The fourth tower of logarithms, $\sum_k c_{k4} a_s^k \ln^{2k-3} N$, yields a very large contribution, e.g., it exceeds the effect of the first three towers by a factor of 1.5 at $x = 0.9$. In order to assess the convergence of the large- N expansion, it is useful to notice that the fifth tower, $\sum_k c_{k5} a_s^k \ln^{2k-4} N$, is fixed by available information (including the parameter g_{02} in eq. (3)), except for the second term $g_{33}(a_s \ln N)^2$ of the function g_3 in eq. (8). As shown in table 1, the impact of the unknown constant g_{33} (of which the order of magnitude can be inferred from $k = 3$) is small at $k \geq 5$. The effect of the fifth tower can thus be estimated, and turns out to be rather moderate, e.g., about 25% at $x = 0.9$. This indicates that four to five towers are sufficient for a valid estimate of the soft-gluon corrections to the quark coefficient functions at large x .

To summarize, we have determined the four leading towers of large- x / large- N soft-gluon logarithms entering the quark coefficient functions in DIS, and estimated the fifth tower. There is evidence that – provided the resummation is applied in N -space, avoiding uncontrolled lower $\ln^l N$ terms due to the Mellin transform (2) – these four to five towers are sufficient for a reliable estimate of the higher-order contributions at large x . These terms should thus be included in future analyses of structure function data at very large x .

Acknowledgements:

The author gratefully acknowledges discussions with S. Catani, which gave rise to the research presented here. It is also a pleasure to thank E. Laenen and W.L. van Neerven for useful comments on the manuscript. This work has been supported by the European Community TMR research network ‘Quantum Chromodynamics and the Deep Structure of Elementary Particles’ under contract No. FMRX-CT98-0194.

References

- [1] A. Bodek and U.K. Yang, Phys. Rev. Lett. **82** (1999) 2467
- [2] S. Kuhlmann, H.L. Lai and W.K. Tung., Phys. Lett. **B409** (1998) 271
- [3] C. Adloff et al., H1 collab., Z. Phys. **C 74**, 191 (1997);
J. Breitweg et al., ZEUS collab., Z. Phys. **C 74**, 207 (1997)
- [4] G. Sterman Nucl. Phys. **B281** (1987) 310;
D. Appel, P. Mackenzie and G. Sterman, Nucl. Phys. **B309** (1988) 259
- [5] S. Catani and L. Trentadue, Nucl. Phys. **B327** (1989) 323; **B353** (1991);
S. Catani, G. Marchesini and B.R. Webber, Nucl. Phys. **B349** (1991) 635
- [6] W.A. Bardeen, A.J. Buras, D.W. Duke and T. Muta, Phys. Rev. **D18** (1978) 3998
- [7] W.L. van Neerven and E.B. Zijlstra, Phys. Lett. **B272** (1991) 127; Nucl. Phys. **B383** (1992) 525
- [8] S. Catani, M.L. Mangano and P. Nason, JHEP **07** (1998) 024
- [9] J. Blümlein and S. Kurth, Phys. Rev. **D60** (1999) 014018
- [10] W.L. van Neerven and A. Vogt, in preparation
- [11] S.A. Larin, T. van Ritbergen, and J.A.M. Vermaseren, Nucl. Phys. **B427** (1994) 41;
S.A. Larin, P. Nogueira, T. van Ritbergen, and J.A.M. Vermaseren, Nucl. Phys. **B492** (1997) 338
- [12] W.L. van Neerven and A. Vogt, hep-ph/9907472, Nucl. Phys. **B** (in press)
- [13] A. Vogt, talk presented at QCDNET 99, Florence (Italy), September 1999
- [14] S. Catani, M.L. Mangano, P. Nason and L. Trentadue, Nucl. Phys. **B478** (1996) 273
- [15] M. Glück, E. Reya and A. Vogt, Z. Phys. **C48** (1990) 471;
A. Vogt, thesis, Dortmund University 1992

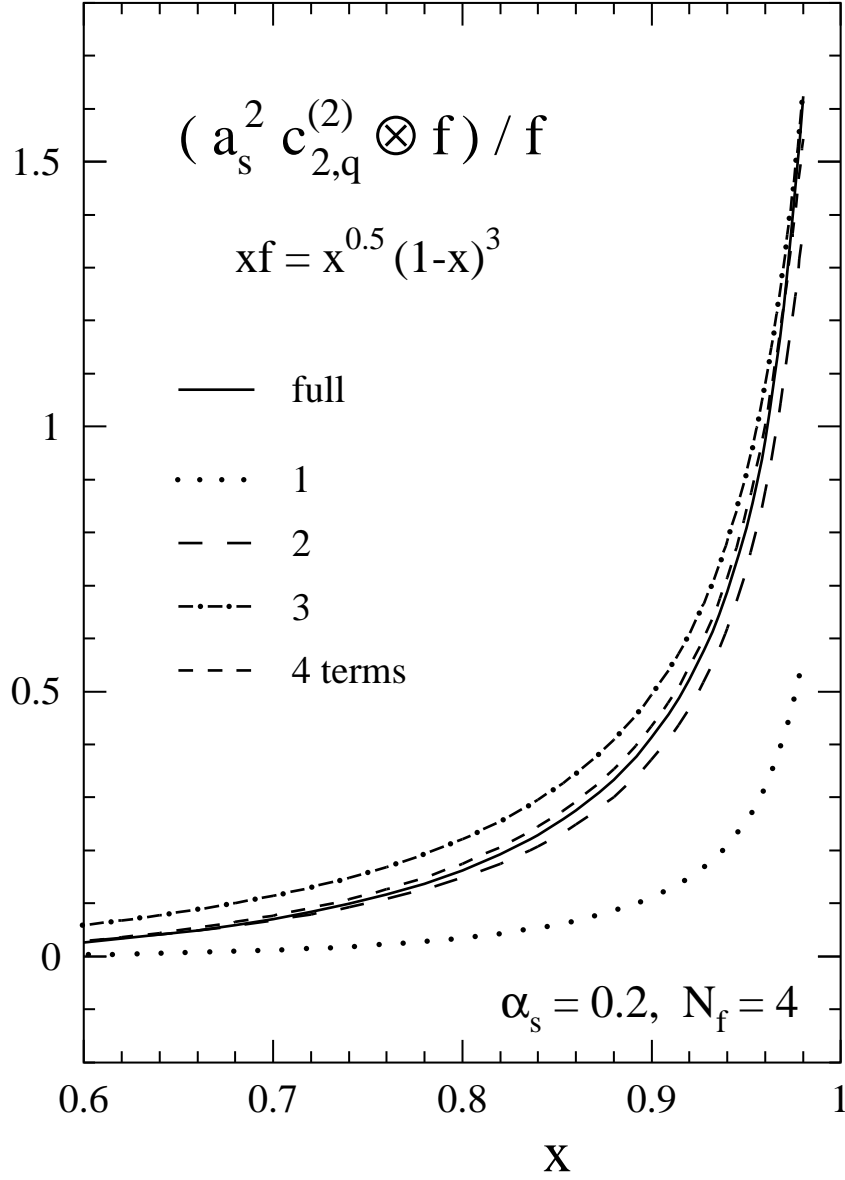


Figure 1: Convolution of the second-order contribution to the coefficient function $C_{2,q}$ with a typical input shape. Shown are the full result [7] for $c_{2,q}^{(2)}$ and four large- N approximations, in which the terms $\sim \ln^4 N$, $\ln^3 N$, $\ln^2 N$ and $\ln N$ in eq. (13) are successively included (e.g., the long-dashed curve includes the $\ln^4 N$ and $\ln^3 N$ parts).

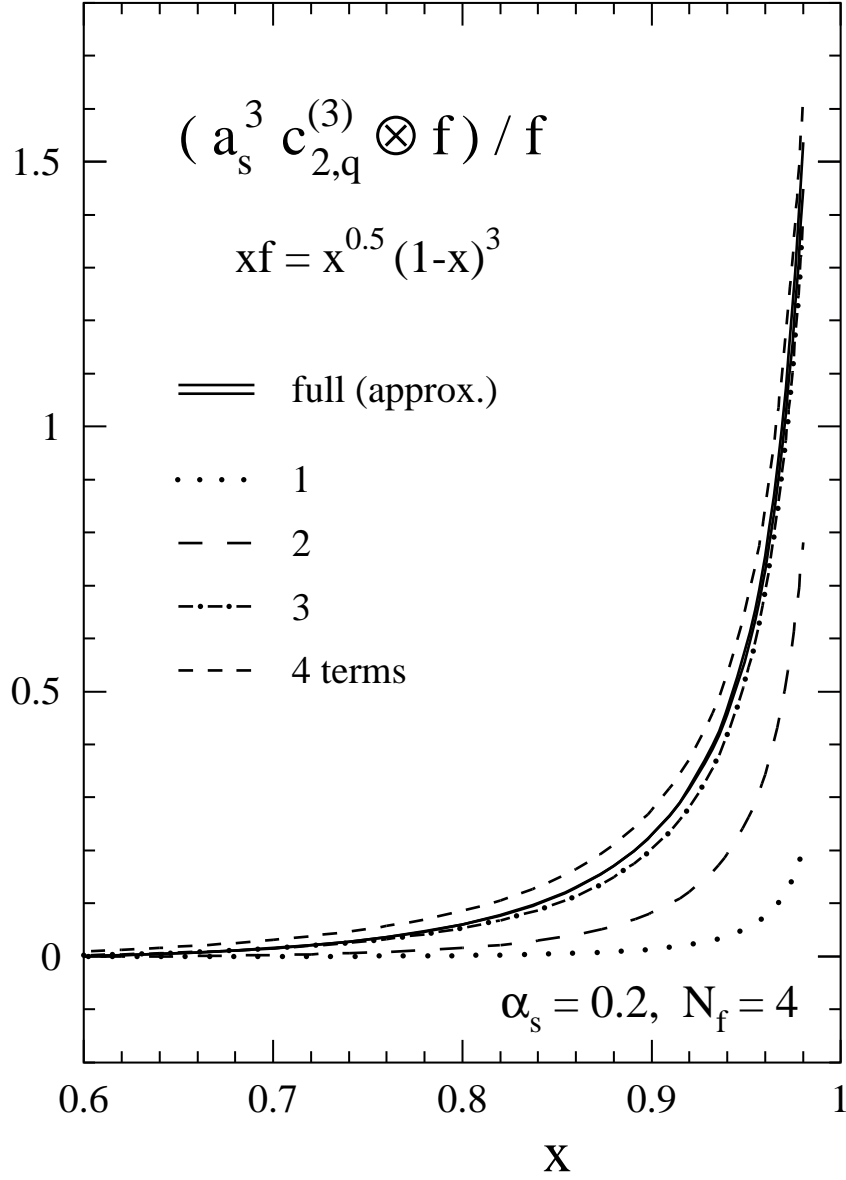


Figure 2: Convolution of the third-order term $c_{2,q}^{(3)}$ of the coefficient function $C_{2,q}$ with a typical input shape. The four leading large- N approximations are compared with a parametrisation [13] of the full result based on the lowest five even-integer moments [11]. The spread of the full curves indicates the residual uncertainty of this parametrisation.

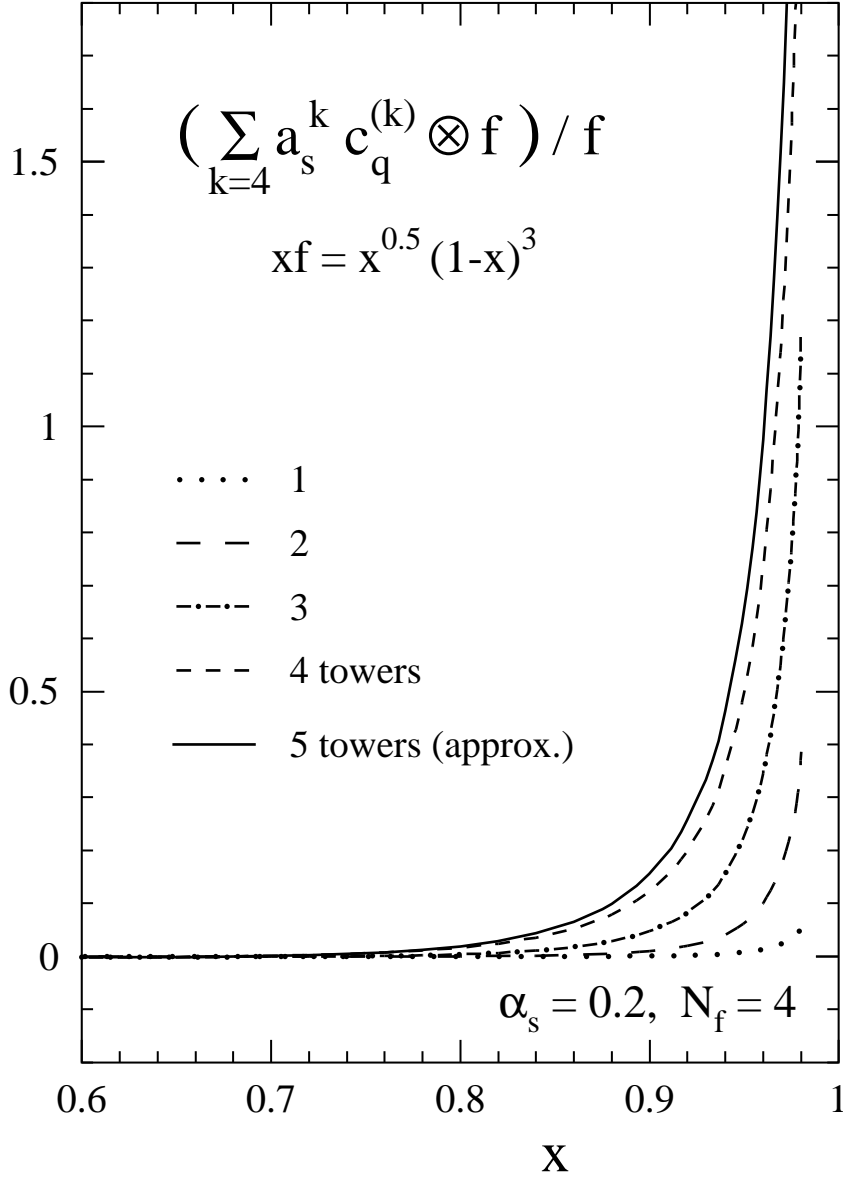


Figure 3: The effect of the higher-order soft-gluon corrections, $\sum_{k=4} \sum_{j=0}^{j_m} c_{kj} a_s^k \ln^{2k-j} N$, to the quark coefficient functions at large N . The results including up to four towers ($j_m = 3$) are derived from the exact coefficients in eq. (14). The impact of the fifth tower ($j_m = 4$) is estimated using $g_{33} = 500$ for the coefficients c_{k5} at $k \geq 5$ in table 1.